

# Constraining holographic technicolor

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## Abstract

We obtain a new bound on the value of Peskin-Takeuchi  $S$  parameter in a wide class of bottom-up holographic models for technicolor. Namely, we show that weakly coupled holographic description in these models implies  $S \gg 0.2$ . Our bound is in conflict with the results of electroweak precision measurements, so it strongly disfavors the models we consider.

**Keywords:** Electroweak symmetry breaking, Holographic duality, Oblique parameters, Tree unitarity.

## 1. Introduction

Long-anticipated discovery [1] of a Higgs-like particle brings us face-to-face with major challenge of comprehending in detail the mechanism of electroweak symmetry breaking [2]. An interesting option potentially leading to composite Higgs is provided by strongly interacting models similar to technicolor [3]. The latter is usually represented by a gauge theory with chiral symmetry  $SU(N_f)_L \times SU(N_f)_R$  which breaks down to  $SU(N_f)_V$  in QCD-like manner;  $N_f$  is the number of techniflavors. Electroweak symmetry is a gauged  $SU(2)_L \times U(1)_Y$  subgroup of the chiral group, so it is broken due to chiral symmetry breaking. Unfortunately, the simplest, literally drawn from QCD technicolor models were ruled out long ago, as they predict unacceptably large values of the Peskin-Takeuchi [4]  $S$  parameter. This leaves open [5] a “walking” version [6] which, however, lacks contact with the phenomenological information accumulated by hadron physics.

The gauge/gravity holographic duality [7] enters at this stage as an approach to studying technicolor models in terms of their weakly coupled gravity duals in five dimensions [8, 9, 10, 11, 12, 13]. Using the holographic dictionary [7], one relates conserved currents  $j_\mu^L, j_\mu^R$  of the left and right  $SU(N_f)$  chiral groups to five-dimensional gauge fields<sup>1</sup>  $L_M$  and  $R_M$ . This promotes the global  $SU(N_f)_L \times SU(N_f)_R$  symmetry of the original model to the gauge symmetry of the holographic dual.

The problem of computing current correlators then reduces to that of solving classical equations for the dual fields.

Since dual descriptions of realistic technicolor theories are unknown (see, however, Refs. [14, 13]), one tends to adopt a bottom-up approach [15] trying to guess the field content and Lagrangian of the five-dimensional dual model on phenomenological grounds. To this end one introduces new fields besides  $L_M$  and  $R_M$ , for instance, an  $SU(N_f)_L \times SU(N_f)_R$  bifundamental scalar  $X$  representing techniquark condensate [8, 10, 11, 12]. One also selects appropriate conditions at the boundaries of the 5D space and allows for departures from the  $AdS_5$  geometry [9, 10, 11]. The price to pay is the absence of an ultraviolet completion of the model which therefore has the status of an effective theory below a certain UV cutoff.

This simple picture is far from being justified in any rigorous sense. Nevertheless, one hopes that such models capture essential features of strongly coupled dynamics and therefore serve as good toy models for technicolor theories.

In this Letter we derive a new constraint on a class of holographic technicolor models, namely, those [10, 11] containing two  $SU(N_f)$  gauge fields  $L_M, R_M$  and bifundamental  $X$ . The fields live in an interval in the warped fifth dimension, with boundary conditions to be specified below. We show that weakly coupled description of these holographic models implies large values of  $S$  parameter. Namely,  $S \gg 0.2$ , otherwise: (i) the UV cutoff drops below  $6\pi m_W/g \sim 2.5$  TeV; (ii) correlators of electroweak currents with momenta exceeding the UV

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<sup>1</sup>Hereafter  $\mu, \nu = 0 \dots 3$  and  $M, N = 0 \dots 3, 5$ .

cutoff are sensitive to strongly coupled sector of the 5D theory and therefore not tractable; (iii) no reliable predictions for the spectrum can be made. Properties (i)—(iii) degrade the status of the holographic technicolor models to that of theories with massive  $W$  bosons and no Higgs mechanism: the latter are also strongly coupled above a few TeV. On the other hand, the constraint  $S \gg 0.2$  is in conflict with the experimental result [16]  $S = -0.07 \pm 0.1$  and therefore strongly disfavors the models.

We introduce the models in Sec. 2, review their spectrum and computation of  $S$  in Secs. 3 and 4, respectively. In Sec. 5 we present a derivation of the weak coupling condition in general warped background. On this basis we obtain new bound on  $S$  in Sec. 6. In Sec. 7 we show that our bound is stable with respect to the addition of higher-order operators to the Lagrangian. We summarize in Sec. 8.

## 2. Models

The models we consider [10, 11] are formulated in a patch of 5D space with warp factor  $w(z)$ ,

$$ds^2 = w^2(z) (\eta_{\mu\nu} dx^\mu dx^\nu - dz^2), \quad z \in [z_{UV}, z_{IR}],$$

where  $w(z_{UV}) = 1$ . The action reads,

$$\mathcal{S} = \int dz d^4x \text{tr} \left[ w(z) (L_{MN}^2 + R_{MN}^2) / 2g_5^2 + w^3(z) D_M X^\dagger D^M X - w^5(z) V(X) \right]. \quad (1)$$

It describes two  $SU(N_f)$  gauge fields  $L_M$  and  $R_M$  interacting with scalar  $X$ ;  $g_5$  is the five-dimensional gauge coupling. Hereafter the integrals over  $z$  run from  $z_{UV}$  to  $z_{IR}$ ; we write  $w(z)$  explicitly and convolve indices with mostly negative flat metric. In our notations  $L_M$  and  $R_M$  are anti-Hermitean matrices,  $L_{MN} = \partial_{[M} L_{N]} + L_{[M} L_{N]}$ . The bifundamental scalar  $X$  is gauge transformed as  $X \rightarrow \omega_L X \omega_R^\dagger$ , where  $\omega_{L,R} \in SU(N_f)_{L,R}$ ; its covariant derivative is  $D_M X = \partial_M X + L_M X - X R_M$ .

We assume that the models (1) are dual to strongly coupled technicolor theories. Then  $SU(N_f)_L \times SU(N_f)_R$  gauge symmetry must be broken to the diagonal subgroup  $SU(N_f)_V$ . To achieve this, we invoke two sources of symmetry breaking that work together [10, 11]. One is the boundary conditions at the IR brane,

$$L_\mu = R_\mu, \quad \partial_z L_\mu = -\partial_z R_\mu \quad \text{at } z = z_{IR}, \quad (2)$$

and another is the vacuum profile of  $X$  which is assumed<sup>2</sup> to have the form  $X_0 = v(z) \cdot \mathbb{I}$ , where  $\mathbb{I}$  is the

<sup>2</sup>The scalar potential  $V(X)$  and boundary conditions for  $X$  should be chosen accordingly.

$N_f \times N_f$  unit matrix,  $v(z)$  is real. The conditions (2) and vacuum  $X_0$  are preserved by the diagonal gauge transformations with  $\omega_L = \omega_R$  and  $\partial_z \omega_L|_{z_{IR}} = \partial_z \omega_R|_{z_{IR}} = 0$ , so the diagonal subgroup  $SU(N_f)_V$  remains unbroken.

We do not consider theories [18, 9] with explicit breaking of gauge invariance in the bulk and accidentally enlarged gauge symmetry at the quadratic level<sup>3</sup>, as these properties generically lead to pathologies: strong coupling at all scales, ghosts, etc.

The models we consider are parametrized by the coupling constant  $g_5$ , warp factor  $w(z)$ , and vacuum profile  $v(z)$ . We note that a subclass of models without the scalar  $X$  [17] is effectively obtained at  $v(z) = 0$ ; gauge symmetry in this case is broken by the boundary conditions (2). Our analysis applies at  $v(z) = 0$  equally well. We impose consistency requirements: (i)  $v^2(z) \ll \Lambda_5^3$ , where  $\Lambda_5$  is a UV cutoff of the models (1); (ii)  $w(z)$  and  $v(z)$  do not vary on the physical length scale of order  $w(z)\Delta z \sim \Lambda_5^{-1}$ . The conditions (i), (ii) ensure the suppression of higher-order operators  $(X^\dagger X)^n / \Lambda_5^{3n}$ ,  $(D_M X D_M X^\dagger)^n / \Lambda_5^{5n}$ , etc., which are present in the general effective Lagrangian.

By construction, the fields  $L_M$  and  $R_M$  are dual to the chiral currents  $j_\mu^L, j_\mu^R$  of the technicolor theory. This means [7] that the current correlators are computed holographically in terms of  $L_M$  and  $R_M$ . First, one solves the classical field equations with the boundary conditions (2) and

$$L_\mu|_{z_{UV}} = \bar{L}_\mu(x), \quad R_\mu|_{z_{UV}} = \bar{R}_\mu(x). \quad (3)$$

Second, one computes the action (1) for the solution, to obtain the functional  $\mathcal{S} = \mathcal{S}[\bar{L}, \bar{R}]$ . In the holographic approach  $\mathcal{S}$  is interpreted as a generating functional [7, 15] for correlators of the chiral currents. In particular,

$$\langle j_\mu^{La}(x) j_\nu^{Rb}(y) \rangle = -i \frac{\delta^2 \mathcal{S}}{\delta \bar{L}^{\mu a}(x) \delta \bar{R}^{\nu b}(y)} \Big|_{\bar{L}=\bar{R}=0}, \quad (4)$$

where the component fields  $L_\mu^a = 2i \text{tr}(L_\mu t^a)$  and  $R_\mu^a$  are introduced. The field content of the four-dimensional technicolor theory remains unknown in the bottom-up holographic approach: the theory is defined by correlators like (4).

To add electroweak interactions, we consider the 4D picture and embed exactly one<sup>4</sup> copy of  $SU(2)_L$  and  $U(1)_Y$  electroweak groups into the left and right  $SU(N_f)$

<sup>3</sup>One can formally restore gauge invariance by introducing Stückelberg/spurion fields [18, 9]. This does not make a theory healthy.

<sup>4</sup>In other models [3] one embeds electroweak group  $N_f/2$  times and obtains  $N_f/2$  times larger value of  $S$  parameter.

chiral groups. We couple the respective isospin components  $j_\mu^{L\bar{a}}$  and  $j_\mu^{R3}$  of the chiral currents to the  $SU(2)_L$  and  $U(1)_Y$  electroweak bosons, where  $\bar{a} = 1 \dots 3$ . This corresponds to gauging  $SU(2)_L \times U(1)_Y$  subgroup of the global flavor group. The electroweak symmetry is then spontaneously broken due to chiral symmetry breaking. We invoke 5D description by noting that electroweak observables are related to the current correlators which, in turn, are computed via Eq. (4). For example, the polarization operator between the  $SU(2)_L$  gauge field and hypercharge field is equal to

$$\begin{aligned} \text{Diagram: } \text{W}_\mu^{\bar{a}} \text{ (wavy line)} \text{ --- } \text{B}_\nu \text{ (wavy line)} \text{ with momentum } \vec{p} \text{ entering the vertex} &= i\eta_{\mu\nu} g g' \Pi_{\bar{a}Y}(p^2) + p_\mu p_\nu \text{ terms} \quad (5) \\ &= -g g' \int d^4x e^{ipx} \langle j_\mu^{L\bar{a}}(x) j_\nu^{R3}(0) \rangle, \end{aligned}$$

where  $g$  and  $g'$  are the electroweak gauge couplings.

### 3. Spectrum

In quest for constraining the models (1) we need information about their spectra. Since the extent of the fifth coordinate is finite, there is a discrete tower of Kaluza-Klein modes which are interpreted as technimesons. Below we analyze vector excitations and leave aside scalars<sup>5</sup> whose spectrum depends on the form of the potential  $V(X)$ .

One notices that the action (1), boundary conditions (2) and vacuum profile  $X_0$  are invariant under  $\mathbb{Z}_2$  parity transformations  $L_M \leftrightarrow R_M$ ,  $X \leftrightarrow X^\dagger$ . Thus, linearized equations for the parity-even vector field  $V_M = (L_M + R_M)/\sqrt{2}$  decouple from equations for the parity-odd axial-vector field  $A_M = (L_M - R_M)/\sqrt{2}$ . In the  $V_5 = A_5 = 0$  gauge, it is consistent to set  $\partial_\mu V^\mu = \partial_\mu A^\mu = 0$ , and the field equations become

$$-\frac{1}{w} \partial_z (w \partial_z V_\mu) - p^2 V_\mu = 0, \quad (6a)$$

$$-\frac{1}{w} \partial_z (w \partial_z A_\mu) - (p^2 - 2g_5^2 w^2 v^2) A_\mu = 0, \quad (6b)$$

where  $p_\mu$  is 4D momentum. Since  $V_\mu$  is the gauge field of the unbroken diagonal subgroup, symmetry-breaking effects due to  $v(z) \neq 0$  are felt only by  $A_\mu$ . We supplement Eqs. (6) with boundary conditions

$$V_\mu|_{z_{UV}} = \partial_z V_\mu|_{z_{IR}} = 0, \quad A_\mu|_{z_{UV}} = A_\mu|_{z_{IR}} = 0, \quad (7)$$

deduced from Eqs. (2) and (3). Equations (6), (7) form two independent boundary value problems for the

vector and axial-vector mass spectra  $p^2 = (m_n^V)^2$  and  $p^2 = (m_n^A)^2$ ; we denote the respective eigenfunctions by  $V_n(z)$  and  $A_n(z)$ . The normalization condition follows from (1), it reads:  $\int dz w(z) V_n(z) V_{n'}(z) = \delta_{nn'}$  and likewise for  $A_n(z)$ .

It is not possible to find the spectra for arbitrary  $w(z)$  and  $v(z)$ . There are some general properties, however. First, the operators in Eqs. (6) and hence eigenvalues  $(m_n^V)^2$ ,  $(m_n^A)^2$  are positive-definite. Second, the axial-vector masses are larger<sup>6</sup>,  $m_n^A \geq m_n^V$ .

One learns more from the vector Green's function

$$G_p^V(z, z') = - \sum_n \frac{V_n(z) V_n(z')}{p^2 - (m_n^V)^2}, \quad (8)$$

which satisfies the boundary conditions (7) for vectors and Eq. (6a) with  $\delta(z - z')/w(z)$  in the right-hand side. One solves these equations at  $p^2 = 0$ ,

$$G_{p=0}^V(z, z') = \theta(z - z') I(z') + (z \leftrightarrow z'), \quad (9)$$

where  $I(z) = \int_{z_{UV}}^z dz' / w(z')$ . Combining Eqs. (8) and (9), one finds a sum rule for the vector masses [10],

$$\int dz w(z) G_{p=0}^V(z, z) = \sum_n \frac{1}{(m_n^V)^2} = \int dz w(z) I(z).$$

This relation sets a bound on the mass  $m_1^V$  of the lightest vector technimeson:

$$\frac{1}{(m_1^V)^2} \leq \int dz w(z) I(z). \quad (10)$$

Since  $m_n^A \geq m_n^V$ , the axial-vector masses are also bounded by the right-hand side of Eq. (10).

The axial-vector Green's function  $G_p^A(z, z')$  is defined in a similar way, as a solution to Eq. (6b) with  $\delta(z - z')/w(z)$  in the right-hand side and boundary conditions (7) for  $A_\mu$ . At  $p^2 = 0$  it can be expressed via a particular solution  $a(z)$  of Eq. (6b) satisfying  $a(z_{UV}) = 1$ ,  $a(z_{IR}) = 0$ . One obtains,

$$G_{p=0}^A(z, z') = \theta(z - z') a(z) a(z') I_A(z') + (z \leftrightarrow z'), \quad (11)$$

where  $I_A(z) = \int_{z_{UV}}^z dz' / [w(z') a^2(z')]$ .

<sup>5</sup>Including the Nambu-Goldstone bosons (technipions) [15].

<sup>6</sup>At  $v(z) = 0$  this is the consequence of the fact that  $A_n(z)$  satisfy the same equation as  $V_n(z)$ , but with the Dirichlet boundary condition at  $z = z_{IR}$  instead of the Neumann one. At  $v(z) \neq 0$  the axial masses are shifted further upwards because the additional term in Eq. (6b) is positive.

#### 4. $S$ parameter

The Peskin-Takeuchi  $S$  parameter [4] measures contributions of new physics to the polarization operator  $\Pi_{3Y}$ ,

$$S = -16\pi \left. \frac{d\Pi_{3Y}}{dp^2} \right|_{p^2=0}. \quad (12)$$

The value of  $S$  is extracted from the electroweak precision measurements.

We evaluate  $S$  by the holographic recipe (4), (5). Equation (4) involves only quadratic part of the action, so we solve linear equations (6) with boundary conditions (2), (3),

$$\begin{aligned} V_\mu(p, z) &= \bar{V}_\mu(p) + p^2 \bar{V}_\mu(p) \int dz' w(z') G_p^V(z, z'), \\ A_\mu(p, z) &= \bar{A}_\mu(p) a(z) \\ &\quad + p^2 \bar{A}_\mu(p) \int dz' w(z') G_p^A(z, z') a(z'), \end{aligned} \quad (13)$$

where  $a(z)$  is defined in the previous section,  $\bar{V}$  and  $\bar{A}$  are the linear combinations of  $\bar{L}$  and  $\bar{R}$ . Upon integrating by parts, one writes for the quadratic part of the action

$$S^{(2)} = \frac{1}{g_5^2} \int d^4x \text{tr} (V_\mu \partial_z V_\mu + A_\mu \partial_z A_\mu) \Big|_{z_{UV}}.$$

We substitute solutions (13) into the action and vary it with respect to  $\bar{L}$ ,  $\bar{R}$ . The result for  $\Pi_{3Y}$  is

$$\begin{aligned} \Pi_{3Y}(p^2) &= \frac{1}{2g_5^2} \partial_z a \Big|_{z=z_{UV}} \\ &\quad - \frac{p^2}{2g_5^2} \int dz' w(z') \partial_z (G_p^V(z, z') - G_p^A(z, z') a(z')) \Big|_{z=z_{UV}}. \end{aligned} \quad (14)$$

We finally compute  $S$  parameter [9, 10, 11]:

$$S = \frac{8\pi}{g_5^2} \int dz w(z) [1 - a^2(z)], \quad (15)$$

where the explicit Green's functions (9), (11) at  $p^2 = 0$  were used. We remind that  $a(z)$  satisfies Eq. (6b) with  $p^2 = 0$  and boundary conditions  $a(z_{UV}) = 1$ ,  $a(z_{IR}) = 0$ .

The first term in Eq. (14) does not depend on  $p^2$  and therefore represents the  $Z$ -boson mass:

$$i\eta_{\mu\nu} g g' \Pi_{3Y}(0) = \text{diagram} = -i\eta_{\mu\nu} \cos \theta_W \sin \theta_W m_Z^2.$$

Here we ignored  $p_\mu p_\nu$ -terms and introduced the weak mixing angle,  $\text{tg } \theta_W = g'/g$ . The expression (14) gives

$$m_W^2 = m_Z^2 \cos^2 \theta_W = -\frac{g^2}{2g_5^2} \partial_z a \Big|_{z_{UV}}, \quad (16)$$

where the first equality is a consequence of the custodial symmetry inherent in the models (1). Non-zero masses of  $W$  and  $Z$  bosons are manifestations of the electroweak symmetry breaking, cf. Refs. [15, 19].

In Ref. [11] it was proven that  $S > 0$  in the class of models we consider. This is seen from Eq. (15): the function  $f(z) = a w \partial_z a$  is negative, since  $\partial_z f > 0$  and  $f(z_{IR}) = 0$  due to Eq. (6b) and  $a(z_{IR}) = 0$ . In other words,  $\partial_z a^2 < 0$ , i.e.  $a^2(z)$  monotonically decreases from  $a^2(z_{UV}) = 1$  to  $a^2(z_{IR}) = 0$  implying  $a^2 < 1$  and  $S > 0$ .

Below we further constrain the value of  $S$  by making use of an additional requirement of weak coupling.

#### 5. Weak coupling condition

The model (1) is non-renormalizable and therefore makes sense below some energy cutoff  $\Lambda_5$ . In flat space-time  $\Lambda_5$  is computed from the partial amplitudes for gauge boson scattering. On dimensional grounds these are proportional to  $g_5^2 P$ , where  $P$  is a 5D momentum. The amplitudes grow with energy and break unitarity bound at  $P \gtrsim 1/g_5^2$  signaling strong coupling. Thus,  $\Lambda_5 \sim 1/g_5^2$ .

In warped spacetime the situation is more subtle [20, 21]. Correlators from the UV brane to UV brane, such as (4), are functions of the conformal momentum  $p$ . On the other hand, scattering at  $z = z_0$  is perturbative if the local physical momentum  $P = p/w(z_0)$  satisfies  $P \ll \Lambda_5$ . Thus, brane-to-brane correlators are completely in the weak coupling regime at  $p \ll \Lambda_5 w_{\min}$ , where  $w_{\min}$  is the minimal value of  $w(z)$ . They can still be tractable at higher momenta if contributions from the strongly coupled region  $w(z) < p/\Lambda_5$  are suppressed.

Let us compute the UV cutoff for the general background  $w(z)$ . This generalizes the analysis of Refs. [17] performed in the case of flat metric. To this end we consider the amplitude  $\mathcal{A}_{nn' \rightarrow mm'}$  for the vector-mode scattering  $V_n^a V_{n'}^b \rightarrow V_m^a V_{m'}^b$ . At the tree level, this amplitude is the sum of a  $V^4$  vertex ( $\times$ ) and exchange diagrams ( $\rightarrow$ ). The vertices  $VVA$  and  $VVX$  are forbidden by parity conservation and  $SU(2)_V$  gauge symmetry, respectively. What remains are the diagrams involving  $V_\mu$  only,

$$V_{n'}^b \text{---} \text{diagram} \text{---} V_{m'}^b = \text{diagram } s + \text{diagram } t + \text{diagram } u + \text{diagram}$$

We calculate the amplitude at high energies when many Kaluza-Klein modes are ultrarelativistic — as well as

the colliding particles. For the latter, we consider longitudinal polarizations  $\epsilon^\mu(p) \approx p^\mu/m$  and isospin states  $(ab) \rightarrow (ab)$ . We obtain<sup>7</sup>,

$$\mathcal{A}_{nn' \rightarrow mm'} = -g_5^2 d^{ab} g_{nn'mm'} \frac{3 + \cos^2 \theta}{2 + 2 \cos \theta}. \quad (17)$$

Here  $\theta$  is the scattering angle,  $d^{ab} = \sum_c (f^{abc})^2$  involves the  $SU(N_f)$  structure constants  $f^{abc}$ ,  $g_{nn'mm'} = \int dz w \phi_n \phi_{n'} \phi_m \phi_{m'}$  is the overlap integral of functions  $\phi_n = \partial_z V_n / m_n^V$ . To understand the meaning of  $\phi_n$ , one performs the gauge transformation which eliminates longitudinal components  $V_\mu^L = i p_\mu V^L$  and induces instead  $V_5 = \partial_z V^L$ . One sees that  $\phi_n$  are the wave functions of the longitudinal modes; they satisfy completeness relation  $\sum_n \phi_n(z) \phi_n(z') = \partial_z \partial_{z'} G_{p=0}^V(z, z') = \delta(z - z')/w(z)$ , where Eqs. (8), (9) were used.

We expect that in terms of conformal momentum, the cutoff depends on  $z$ . To see this explicitly, we localize colliding particles in the fifth dimension by considering the Kaluza-Klein state  $|V_{z_0}\rangle = \mathcal{N} \sum_{n < n_0} \phi_n(z_0) |V_n\rangle$ , where  $\mathcal{N}$  is a normalization constant. At  $n_0 \gg 1$ , the wave function of this state is concentrated near  $z = z_0$ , as the completeness of  $\phi_n$  suggests. Such a localization is consistent with the presence of the UV cutoff, since, as we pointed out in Sec. 2, the function  $w(z)$  does not strongly vary on the physical distance scale  $\Lambda_5^{-1}$ . The amplitude of the process  $V_{z_0} V_{z_0} \rightarrow V_{z_0} V_{z_0}$  is

$$\mathcal{A}_{z_0} = \mathcal{N}^4 \sum_{nn'mm' < n_0} \phi_n^{(z_0)} \phi_{n'}^{(z_0)} \phi_m^{(z_0)} \phi_{m'}^{(z_0)} \mathcal{A}_{nn' \rightarrow mm'}, \quad (18)$$

where  $\phi_n^{(z_0)} = \phi_n(z_0)$ .

Let us now recall the unitarity conditions  $|\text{Re } \mathcal{A}_l| \leq 1/2$  for partial amplitudes, where  $l$  is the angular momentum. Particularly useful is the constraint

$$|\text{Re } (\mathcal{A}_0 + \mathcal{A}_1)| \equiv \frac{1}{32\pi} \left| \int_{-1}^1 d\cos\theta (1 + \cos\theta) \text{Re } \mathcal{A} \right| \leq 1$$

where the left-hand side is free of collinear divergences. Making use of Eqs. (17), (18) and explicitly writing  $g_{nn'mm'}$ , we find

$$|\text{Re } (\mathcal{A}_0 + \mathcal{A}_1)|_{z_0} = \frac{5g_5^2}{48\pi} d^{ab} \mathcal{N}^4 \sum_{nn'mm' < n_0} \phi_n^{(z_0)} \phi_{n'}^{(z_0)} \times \phi_m^{(z_0)} \phi_{m'}^{(z_0)} \int dz w(z) \phi_n^{(z)} \phi_{n'}^{(z)} \phi_m^{(z)} \phi_{m'}^{(z)} \leq 1. \quad (19)$$

<sup>7</sup>Calculations simplify in the  $\mathcal{R}_\xi$  gauge [22] where the longitudinal components of massive vector modes can be traded at high energies for Nambu-Goldstone bosons.

We consider the indices  $(ab)$ ,  $a \neq b$  belonging to the  $SU(2)$  subgroup of  $SU(N_f)$  and obtain  $d^{ab} = 1$ . One sum in Eq. (19) is proportional to  $\delta(z - z_0)$  due to completeness of  $\phi_n$ , the others are equal to the semiclassical density of states  $\sum_{n < n_0} \phi_n^2(z_0) \approx \Delta P_z / 2\pi = m_{n_0}^V / [\pi w(z_0)]$ . The normalization factor of  $|V_{z_0}\rangle$  equals  $\mathcal{N}^2 = \pi w(z_0) / m_{n_0}^V$ . One sees that the inequality (19) takes the form  $5g_5^2 m_{n_0}^V \leq 48\pi^2 w(z_0)$ . It bounds the value of the highest available mass  $m_{n_0}^V$  and hence conformal momentum:  $p < w(z_0) \Lambda_5$ , where  $\Lambda_5 = 48\pi^2 / 5g_5^2$  is the local scale of strong coupling.

Common sense suggests that theories with too low UV cutoff are not viable. In the rest of this section we argue that the model (1) is not tractable unless

$$m_1^V \ll \Lambda_5 w_{\min}, \quad \text{where} \quad \Lambda_5 = 48\pi^2 / 5g_5^2. \quad (20)$$

Here  $m_1^V$  is the lowest vector mass.

First, one notices that the tower of vector modes is strongly coupled whenever Eq. (20) is violated. Indeed, all vector masses are then above the cutoff in the region  $w(z) < m_1^V / \Lambda_5$ . Mode amplitudes are large there: a semiclassical estimate gives  $\phi_n^2(z)$ ,  $V_n^2(z) \propto 1/w(z)$ . Thus, processes involving vector modes receive large contributions from the strongly coupled region  $w(z) < m_1^V / \Lambda_5$  and cannot be treated within the effective theory (1). This prevents one to draw any conclusions about vector technimesons and hence damages predictability.

In warped models, one can sometimes consistently consider conformal momenta exceeding  $\Lambda_5 w_{\min}$ , as long as one deals exclusively with brane-to-brane correlators [20, 21]. The point is that at high Euclidean momenta, the brane-to-bulk propagator decays as  $\exp[-p(z - z_{UV})]$ , which can suppress effects coming from the strongly coupled region  $w(z) < p / \Lambda_5$ . For  $p \sim \Lambda_5 w_{\min}$  such suppression mechanism requires  $\Lambda_5 w_{\min} (z_{IR} - z_{UV}) \gg 1$ . This, in turn, implies the inequality (20), since  $m_1^V (z_{IR} - z_{UV}) \simeq \pi/2$  according to the Bohr-Sommerfeld rule. On the contrary, once the inequality (20) is violated,  $\Lambda_5 w_{\min}$  is the true cutoff for momenta  $p$  referring to the UV brane.

Another way to see the strong coupling problem for the brane-to-brane correlators at  $m_1^V > \Lambda_5 w_{\min}$  is to consider the propagator in the form (8). At  $p \lesssim m_1^V$  it is dominated by the first term in the sum (8) and therefore proportional to  $V_1(z)$ . The latter grows with  $z$ , as the lowest eigenfunction of Eqs. (6a), (7). This means that  $G_p^V$  cannot suppress contributions from the strongly coupled region  $w(z) < p / \Lambda_5$  for momenta in the range  $\Lambda_5 w_{\min} < p < m_1^V$ .

So far we have argued that once the inequality (20) is violated, the theory makes sense only at  $p < \Lambda_5 w_{\min}$ .



Let us show that the scale  $\Lambda_5 w_{\min}$  is unacceptably low, even somewhat lower than the cutoff in a 4D theory of massive  $W$ -bosons without the Higgs mechanism. To this end we use the Rayleigh-Ritz inequality for the lowest eigenvalue of Eqs. (6a), (7),

$$(m_1^V)^2 \leq \frac{\int dz w (\partial_z f)^2}{\int dz w f^2},$$

which holds for arbitrary function  $f(z)$  satisfying  $f(z_{UV}) = 0$ . We select  $f(z) = a(z) - 1$ , where  $a(z)$  enters Eq. (16). Since in the case under consideration  $\Lambda_5 w_{\min} \lesssim m_1^V$ , we have

$$\Lambda_5^2 w_{\min}^2 \lesssim \frac{\int dz w (\partial_z a)^2}{\int dz w (a-1)^2}. \quad (21)$$

Integrating by parts and using Eq. (6b) at  $p^2 = 0$ , one shows that the numerator in Eq. (21) is smaller than  $-\partial_z a|_{z_{UV}}$ . The denominator equals  $\int (a-1)^2 da (w^2/wa') \geq -w_{\min}^2/3\partial_z a|_{z_{UV}}$ , where we minimized the term in the parenthesis and then evaluated the integral. One obtains  $\Lambda_5^2 w_{\min}^4 < 3(\partial_z a)_{z_{UV}}^2 = 3[96\pi^2 m_W^2/5\Lambda_5 g^2]^2$ , where Eqs. (16), (20) were used to express  $\partial_z a|_{z_{UV}}$  and  $g_5$ . We get finally  $\Lambda_5 w_{\min} < 6\pi m_W/g$  which proves the statement.

To summarize, the inequality (20) should be valid, otherwise the theory is no better than a 4D theory of massive  $W$ -bosons without the Higgs mechanism.

## 6. Constraint on the $S$ parameter

At the culmination of this Letter we derive a bound on  $S$  parameter from the weak coupling condition (20). First, we show that the  $S$  parameter, Eq. (15), is minimal at  $v(z) = 0$ . To this end we find the variation  $\delta a^2(z)$  due to  $\delta v^2(z) > 0$  by varying and solving Eq. (6b) at  $p^2 = 0$ ,

$$a(z)\delta a(z) = -2g_5^2 \int dz' a(z) G_{p=0}^A(z, z') a(z') \times w^3(z') \delta v^2(z') < 0, \quad (22)$$

where the integrand is positive in virtue of Eq. (11). Thus,  $a^2$  decreases and  $S$  grows as  $v^2$  increases.

At  $v = 0$  we explicitly find  $a(z) = 1 - I(z)/I(z_{IR})$  by solving Eq. (6b) at  $p^2 = 0$ . Substituting this into Eq. (15), we get

$$S > \frac{8\pi}{g_5^2} \int dz \frac{wI}{I(z_{IR})} \geq \frac{8\pi}{g_5^2 I(z_{IR}) m_1^V} \left[ \int dz wI \right]^{1/2} \quad (23)$$

where we took into account  $I(z) < I(z_{IR})$  in the first inequality and Eq. (10) in the second. The integral in

brackets is equal to  $\int I dI w^2 \geq w_{\min}^2 I^2(z_{IR})/2$ . Using Eq. (20), we obtain

$$S > \frac{8\pi w_{\min}}{g_5^2 m_1^V \sqrt{2}} = \frac{5}{6\pi \sqrt{2}} \cdot \frac{\Lambda_5 w_{\min}}{m_1^V} \gg 0.2, \quad (24)$$

in obvious conflict with the experimental data.

## 7. Higher-order operators

The model (1) is defined modulo higher-order terms in the Lagrangian suppressed by the cutoff  $\Lambda_5$ . One asks whether they can alleviate our bound on  $S$ , given the consistency requirements of Sec. 2:  $v^2 \ll \Lambda_5^3$ , and  $w(z)$ ,  $v(z)$  are nearly constant on the length scale  $w(z)\Delta z \sim \Lambda_5^{-1}$ . To this end, let us consider explicitly the lowest of these terms [11],

$$\Delta S = -\frac{c}{\Lambda_5^3} \int dz d^4x \frac{w}{2g_5^2} \text{tr} [L_{MN} X R_{MN} X^\dagger], \quad (25)$$

where  $c \lesssim 1$ ; parity-even terms of the same order are irrelevant as they can be absorbed at the quadratic level into redefinition of  $w(z)$  and  $v^2(z)$ . The correction (25) changes Eqs. (6); in particular, Eq. (6b) becomes

$$-\frac{1}{\tilde{w}} \partial_z (\tilde{w} \partial_z A_\mu) - (p^2 - 2g_5^2 \tilde{w}^2 \tilde{v}^2) A_\mu = 0, \quad (26)$$

where  $\tilde{w} = w(1 + cv^2/2\Lambda_5^3)$  and  $\tilde{v}^2 = v^2 w^3/\tilde{w}^3$  contain small corrections. One calculates  $S$  and obtains,

$$S = \frac{8\pi}{g_5^2} \int dz \tilde{w}(1 - \tilde{a}^2) - \frac{5c}{6\pi \Lambda_5^2} \int dz w v^2 = S_+ + S_-,$$

where  $S_+$  and  $S_-$  are the first and second integral, respectively;  $\tilde{a}(z)$  satisfies Eq. (26) at  $p^2 = 0$  with boundary conditions  $\tilde{a}(z_{UV}) = 1$ ,  $\tilde{a}(z_{IR}) = 0$ .

Off hand, the term  $S_-$  could lower the value of  $S$  parameter. Let us prove, however, that  $|S_-| \ll |S_+|$ . At  $v^2 = 0$  we have  $S_- = 0$ ,  $S_+ \neq 0$ . Let us consider the variation  $\delta v^2(z)$  keeping  $\delta \tilde{w}(z) = 0$ . The variation  $\delta \tilde{a}(z)$  is again given by Eq. (22) with  $w$  and  $v$  replaced by  $\tilde{w}$  and  $\tilde{v}$ . The integrand in Eq. (22) is positive and stays constant on the length scale  $w(z)\Delta z \sim \Lambda_5^{-1}$ . Therefore,

$$-\tilde{a}(z)\delta \tilde{a}(z) \gg 2g_5^2 \Delta z \tilde{a}^4(z) \tilde{I}_A(z) \tilde{w}^3(z) \delta \tilde{v}^2(z),$$

where we used Eq. (11) for the Green's function. This gives

$$\delta S_+ \gg \frac{32\pi}{\Lambda_5^2} \int dz \tilde{w}(z) \tilde{a}^2(z) \delta v^2(z), \quad (27)$$

where we substituted  $\tilde{I}_A(z) \gg \Delta z / [\tilde{w}(z)\tilde{a}^2(z)]$  and ignored small multiplicative correction to  $w(z)$ . We see that as  $v^2$  increases,  $v^2 \rightarrow (1 + \epsilon)v^2$ , the term  $S_+$  grows faster than  $|S_-|$  unless

$$\int dz \tilde{w} \tilde{a}^2 v^2 \ll \int dz w v^2, \quad (28)$$

On the other hand, if the inequality (28) is satisfied, then

$$S_+ > \frac{8\pi}{g_5^2} \int dz \tilde{w} \frac{v^2}{v_{\max}^2} [1 - \tilde{a}^2] \approx \frac{\Lambda_5^3}{c v_{\max}^2} |S_-| \gg |S_-|,$$

where we inserted  $v^2/v_{\max}^2 < 1$  in the integrand, ignored the second term in brackets due to Eq. (28) and expressed the result in terms of  $S_-$ .

So, we proved that  $S \approx S_+$ . The analysis of Sec. 6 goes through for  $S_+$  and yields the bound (24). In this way we come to the intuitively clear conclusion that the higher-order term (25) does not affect this bound (see Ref. [11] for the limited numerical analysis of the same problem). Operators of even higher orders should be even less important.

## 8. Conclusions

In this Letter we derived the weak coupling condition  $m_1^V \ll \Lambda_5 w_{\min}$ , where  $m_1^V$  is the lowest Kaluza-Klein mass,  $\Lambda_5 w_{\min}$  is the redshifted cutoff. We proved that within the holographic technicolor models defined by Eqs. (1), (2), this condition bounds the value of  $S$  parameter,  $S \gg 0.2$ , in conflict with experimental data. The latter bound is stable with respect to higher-order corrections and agrees with the conjectured general constraint of Refs. [23].

One can interpret our results in terms of 4D technicolor theory by recalling that  $S \propto 1/g_5^2 \propto N_c$ , where  $N_c$  is the number of technicolors. Thus,  $S$  is smallest at  $N_c \sim 1$  when holography is not trustworthy. The opposite requirement of weakly coupled holographic description leads to a lower bound on  $S$  which, as we demonstrated, is  $S \gg 0.2$ .

Since the troubles come from vectors and axial vectors, our bound can possibly be avoided in models with modified vector sectors. One can think of changing the boundary conditions (2) [8, 12] or considering parity breaking [12].<sup>8</sup> In any case healthy models should be special, as our results suggest. If constructed, they would shed light on the structure of phenomenologically viable technicolor theories.

<sup>8</sup>One can also lower  $S$  by loop contributions from extra fermions [24].

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